# Advanced Proof Viewing in PROOFTOOL

## UITP 2014

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# Introduction

# Problem Description

## Context

- Full sequent calculus formalizations of mathematical proofs are large
- Often defined schematically i.e. contains structural regularities
- Proof analysis applies several transformations:
  e.g.: skolemization, cut-elimination, Herbrand formula extraction

# Problem Description

## Question occurring during analysis

- Are there easily removable cuts?
- Which strong quantifier rule is related to which skolem symbol?
- How do skolem symbols propagate through the proof?
- How do we identify individual inferences from the specification after several proof transformations?

# Problem Description

## Approach

- Proof visualization helps to answer these questions
- Original Gentzen notation alone is insufficient
- Use a second view (Sunburst Tree) to compensate the drawbacks
- Implementation for GUI Prooftool

# Gentzen Notation

## Pros of the Gentzen notation

- Detailed low-level view
- Directly reflects the calculus
- Widespread & easy to understand

## Cons of the Gentzen notation

- Focus on single inferences
- Large contexts obscure the inference information
- Layout grows mainly horizontally (empirically width:height $\approx$ 10:1)
- Locating / tracing inferences is hard

# Small example

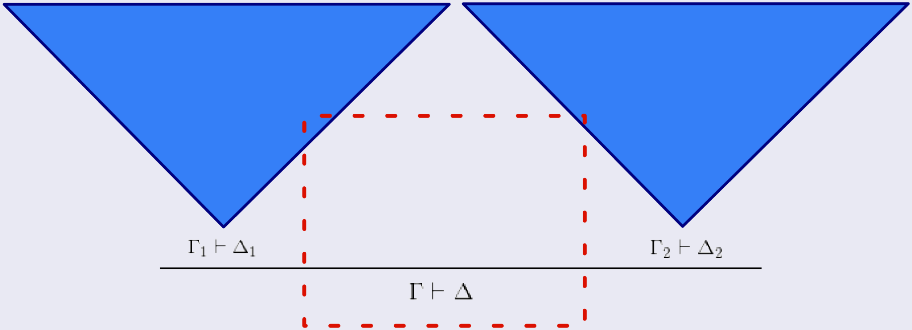
## Commutativity

- Axioms: $0 + 1 = 1$, $1 + 0 = 1$, $x + (y + z) = (x + y) + z$
- We prove: $x + y = y + x$
- 3 instances of Induction schema in antecedent of end-sequent
  IND: $S(0) \wedge \forall x(S(x) \rightarrow S(x + 1)) \rightarrow \forall x S(x)$
- ca. 70 inferences, mostly equational reasoning, 1 cut

# Problem Description

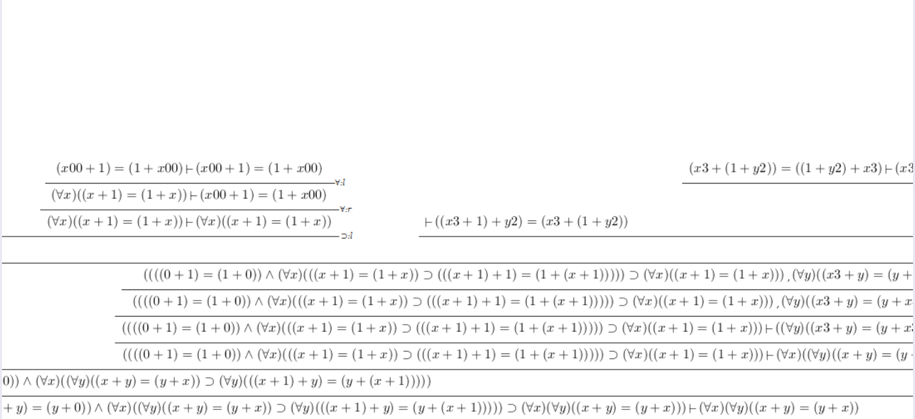
## Drawbacks of the Gentzen notation (1)

Wide proofs leading into binary rules prevent display of auxiliary formulas and primary formulas on the same screen



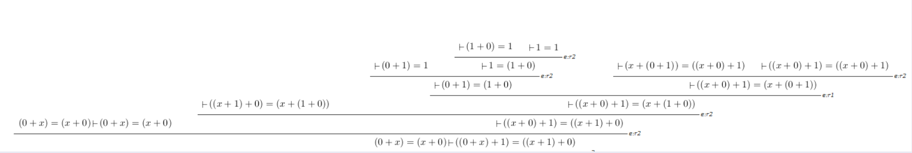
## Drawbacks of the Gentzen notation (2)

Context causes wide horizontal gaps in binary rules



## Drawbacks of the Gentzen notation (3)

"Pseudo-linear" reasoning (trivial chains of $(\rightarrow: l)$ or $(=: l)/(=: r)$ ) grows to the side



# Criteria

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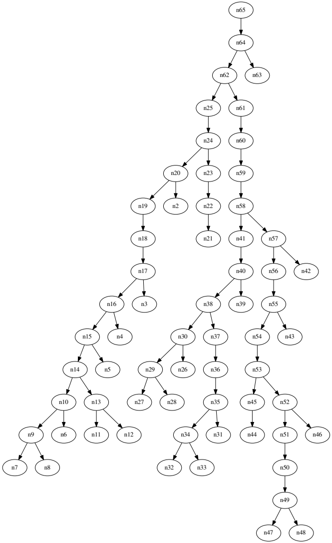
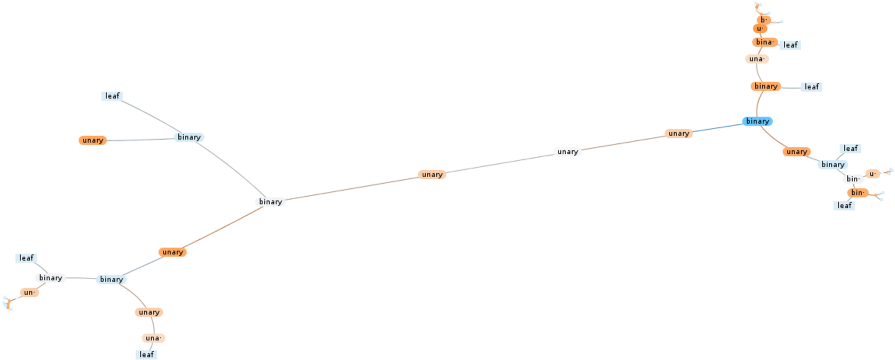
- Displaying of large proofs
- Distinguishing between different kinds of rules
- Easy navigation
- Traceable formula ancestors
- Focus on different aspects of the proof
- Draw information from the structural shape of the proof

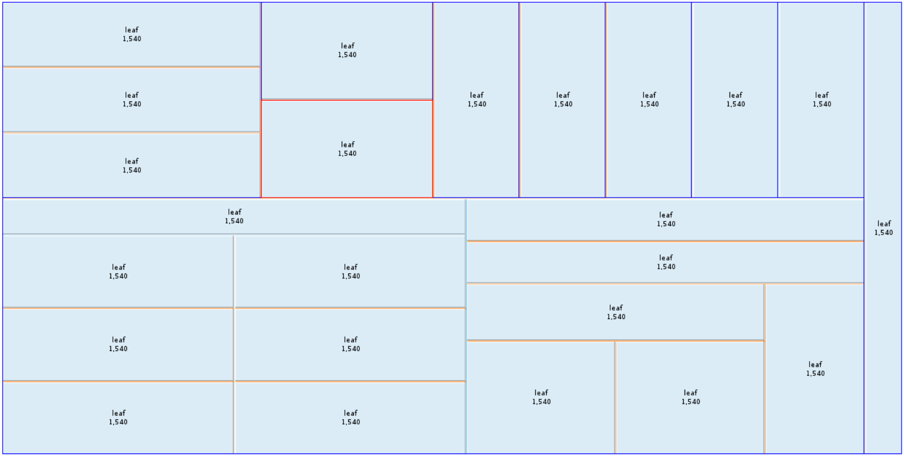
# Alternatives: GraphViz



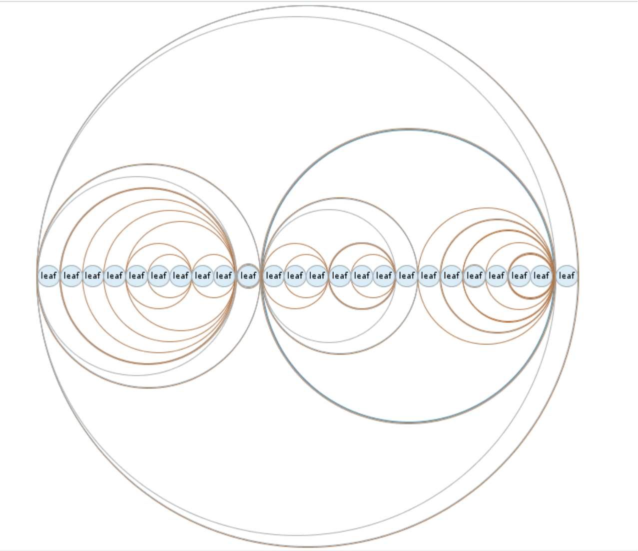
# Alternatives: Hyperbolic Tree



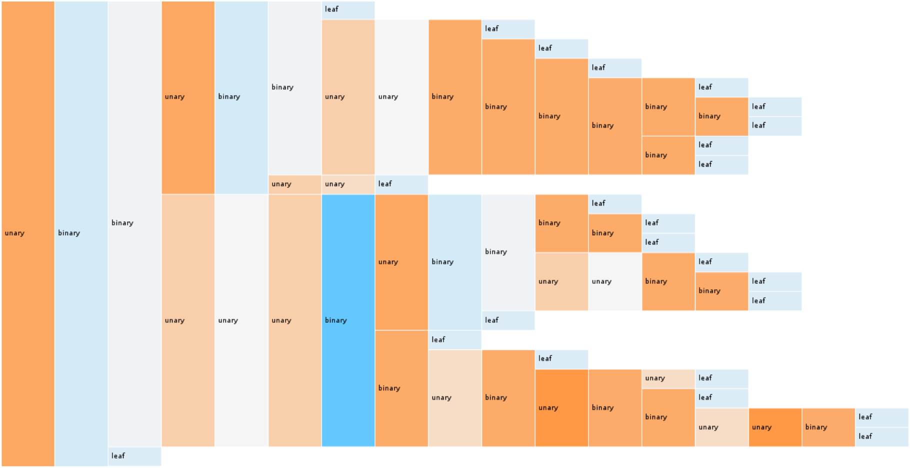
# Alternatives: Rectangular Treemap



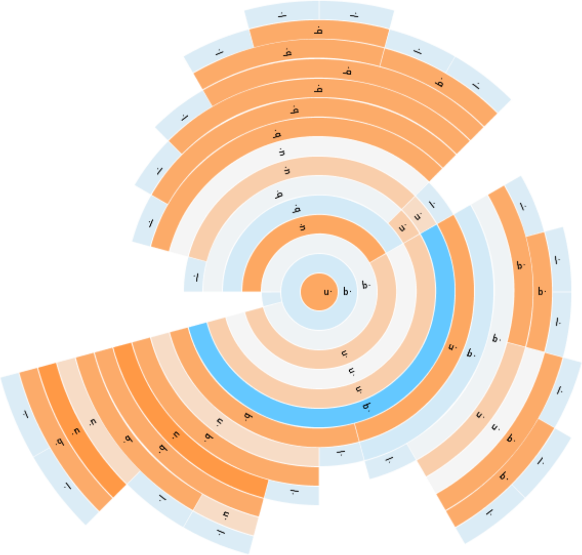
# Alternatives: Circular Treemap



# Alternatives: Icicle Tree



# Alternatives: Sunburst

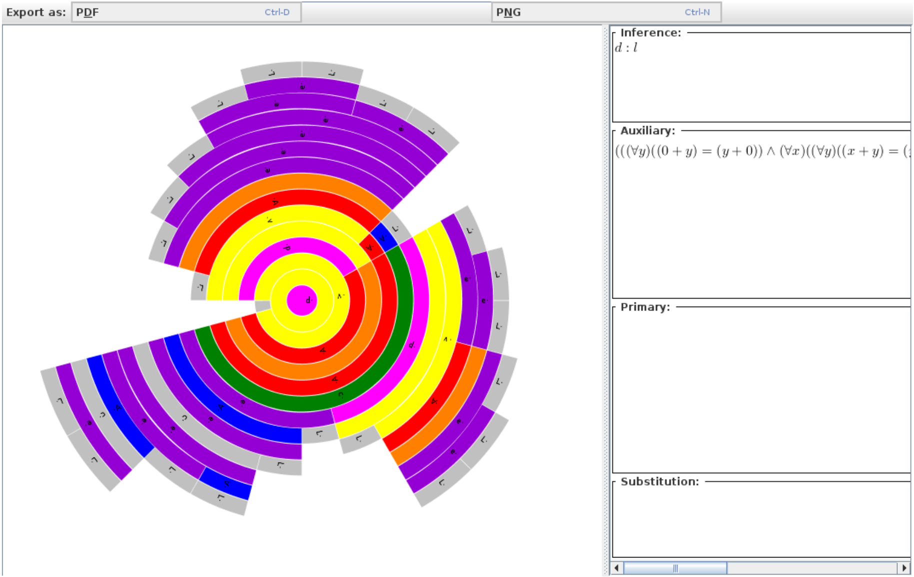


# Combination with Inference view

## Inference View

- Necessary because sequents are invisible in Sunburst View
- Shows primary and auxiliary formulas
- Room for additional information (quantifier instantiations, etc.)

# Sunburst within GAPT



# Conclusion

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- The Gentzen layout needs supporting viewers to handle large proofs better
- Sunburst Trees + Single inference information can provide complementary information

# Open Questions

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- Criteria definition is empirical – what other requirements are there in different contexts?
- Sunburst View works well with trees – what about DAGs?
  Degenerative example: refutation of
  $\{\vdash P(0);\ P(f^n(y)) \vdash;\ P(z) \vdash P(f(z))\}$
- How do we improve navigation in face of information loss by transformations?

# The End

Thanks for your attention!