

Instantiation for Theory Reasoning in Vampire

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Simplifying a Clause from Theory Axioms

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Simplifying a Clause from Theory Axioms

- 1 **initial** $14x \neq x^2 + 49 \vee p(x)$
- 2 **axiom** $x = (x - 1) + 1$
- 3 **1 + 2** $14 \cdot x \neq ((x - 1) + 1) \cdot x + 49 \vee p(x)$
- 4 **axiom** $(x + 1) \cdot y = x \cdot y + y$

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- | | | |
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| 4 | axiom | $(x + 1) \cdot y = x \cdot y + y$ |
| 5 | 3 + 4 | $14 \cdot x \neq ((x - 1) * x + x) + 49 \vee p(x)$ |
| | | ... |

Simplifying a Clause from Theory Axioms

1	initial	$14x \neq x^2 + 49 \vee p(x)$
2	axiom	$x = (x - 1) + 1$
3	1 + 2	$14 \cdot x \neq ((x - 1) + 1) \cdot x + 49 \vee p(x)$
4	axiom	$(x + 1) \cdot y = x \cdot y + y$
5	3 + 4	$14 \cdot x \neq ((x - 1) * x + x) + 49 \vee p(x)$
		...
(much later)		$p(7)$

Alternative: Instantiation

Suppose we guess $x = 7$:

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$$98 \neq 98 \vee p(7)$$

evaluate

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$$98 \neq 98 \vee p(7)$$



$$p(7)$$

evaluate

remove trivial inequality

Alternative: Instantiation

- Find instance that makes theory part of a clause false
- Substitute and delete theory part
- Rule

$$\frac{P \vee D}{D\theta} \text{ theory instance}$$

- P pure (all constant symbols have a fixed interpretation)
- $P\theta$ unsatisfiable in the theory

Alternative: Instantiation

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 - $\theta = \{x \mapsto 7\}$

Alternative: Instantiation

- Why pure?
 - ⇒ We pass $\neg P$ to an SMT solver!
- $\neg P$ has a model: construct θ from model
 - $14x = x^2 + 49$ has a model for $x = 7$
 - $\theta = \{x \mapsto 7\}$
- Model construction needs purity (for now)

Abstraction

- Suppose we want to resolve

$$r(14y)$$

$$\neg r(x^2 + 49) \vee p(x)$$

\Rightarrow No pure literals

Abstraction

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$$r(14y)$$

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- Abstract to

$$z \neq 14y \vee r(z)$$

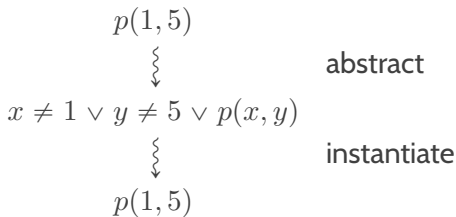
$$u \neq x^2 + 49 \vee \neg r(u) \vee p(x)$$

Problems with Abstraction

- Eager application too expensive, fold into unification

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- Eager application too expensive, fold into unification
- Instantiation undoes abstraction:



Trivial Literals

- Form: $x \neq t$ (x not in t)
- Pure
- x only occurs in other trivial literals or other non-pure literals

Updated Rule

$$\frac{P \vee D}{D\theta} \text{ theory instance}$$

- $P\theta$ unsatisfiable in the theory
- P pure
- P does not contain trivial literals

Improvements to Vampire

SMT-LIB

<i>Logic</i>	<i>New solutions</i>	<i>Uniquely solved</i>
<i>ALIA</i>	<i>1</i>	<i>0</i>
<i>LIA</i>	<i>14</i>	<i>0</i>
<i>LRA</i>	<i>4</i>	<i>0</i>
<i>UFDTLIA</i>	<i>5</i>	<i>0</i>
<i>UFLIA</i>	<i>28</i>	<i>14</i>
<i>UFNIA</i>	<i>13</i>	<i>4</i>

TPTP

<i>Category</i>	<i>New solutions</i>	<i>Uniquely solved</i>
<i>ARI</i>	<i>13</i>	<i>0</i>
<i>NUM</i>	<i>1</i>	<i>1</i>
<i>SWW</i>	<i>3</i>	<i>1</i>

Future Work

- What about multiple solutions?
- What about inequalities ($9 < x^2$ vs. $9 \neq x^2$)
- What about uninterpreted symbols?

Vampire at <https://github.com/vprover/vampire>

Thanks!